# INCORPORATION OF GLIDE AND CREEP MEASUREMENTS INTO SNOW SLAB MECHANICS

C. B. Brown, R. J. Evans, and D. McClung

ABSTRACT

The conventional field measurements in slab avalanche control include snow strength, creep, and glide. The snow strength appears naturally in any failure criterion, but the inclusion of creep and glide data into the slab mechanics is not obvious. Here, the physical features of creep and glide are discussed. This leads to possible models which can be incorporated into a continuum theory. As a result, definite suggestions are made concerning the range of parameters to be measured in the field.

## Introduction

The solution of problems in snow slab mechanics can be based on the formulation of Perla and LaChapelle (1970) where some metamorphosis of either the snow ground interface or some surface in the snow slab causes a reduction in shear capacity. This approach has been further developed by Brown, Evans, and LaChapelle (1972) to find the state of stress in fallen snow and the dimensions of slab avalanches; this work provides no causality for the shear degeneration, and the boundary condition on the interface is described as zero normal motions and a definite shear stress. The solution used is linear elasticity theory.

The first shortcomings of the Brown, Evans, and LaChapelle work are: (1) The inability to model actual interface boundary conditions associated with definite metamorphosis; and (2) the inability to account for the nonlinear and temporal response of the snow. These two features are often included in the expressions, glide and creep. The object of this paper is to discuss the inclusion of these features into snow slab mechanics.

## Glide

The relative motion between the ground surface and the juxtaposed snow will serve as a definition of glide. The motion is the measured translations over definite periods of time. It is natural to think of the onset of and subsequent motion being controlled by the laws of Amontons. These would mean that glide would not occur when

$$\tau < \sigma \, \mu_s \tag{1}$$

where $\tau$ is the interface shear, $\sigma$ the interface normal pressure, and $\mu_s$ the static coefficient of friction. When [1] becomes an equality, glide commences and the resistance to this motion is a shear

$$\tau = \sigma \, \mu_k \tag{2}$$

where $\mu_k$ is kinetic coefficient of friction and $\mu_k < \mu_s$. One consequence of such laws is the monotonic increase of the glide speed with time. Examination of the results of Gand and Zupancic (1965) for glide near Davos indicates that the speed increases with time until a terminal value is attained. This speed is then maintained for the rest of the season. The results in figure 1 have a reverse trend inasmuch as the terminal speed is the decay from the high values of the earlier season. Clearly these two sets of readings from Davos and the Cascades are in conflict. However, they do refute the restraint equation [2]; the speed does stabilize in both cases, and the resistance must depend on the speed. Thus

$$\tau = f(v) \tag{3}$$

where $v$ is the glide speed. The functional form of [3] will apparently be completely different in the two cases discussed.

The small scale features at the interface consist of soft snow laid on a hard ground with the possibility of organic separation. The ground is rugged and, because of the differences between hardness of the ground and snow, it is unlikely that the usual concept of slipping can apply. Rowe (1964) has shown, for materials with very different hardness, failure is by shear in the softer material at a layer beyond the envelope of the rough surface. This means that failure
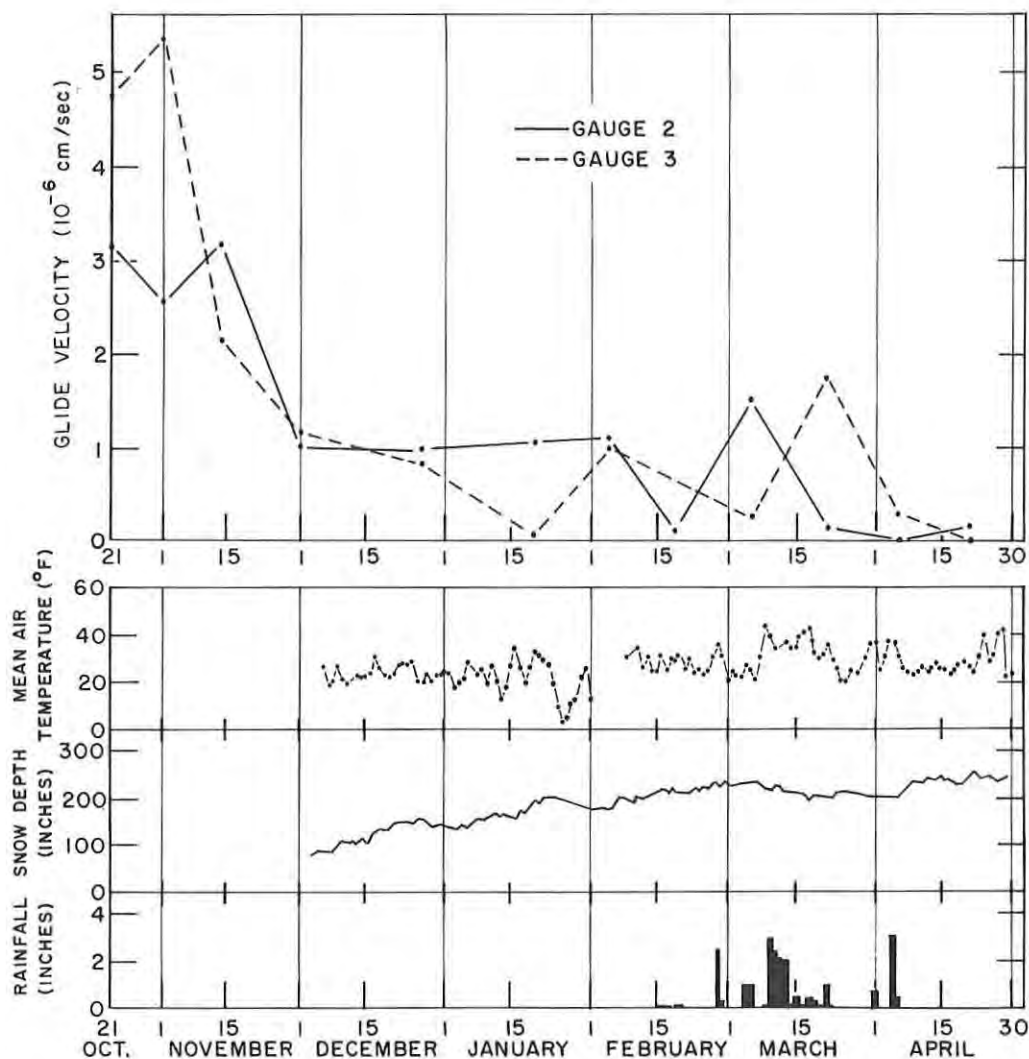
*Figure 1. Glide measurements, 1971-72, Mt. Baker, Washington.*

would be in the snow itself unless the organic layer was smooth and slippery or unless melt water appeared at the interface. Following up on Rowe's work, we would anticipate that the initiation of glide would be associated with the shear capacity of the snow. Here we would consider a sintered material with the conditions ripe for the application of the Bowden-Tabor (1958) theory of failure across the cohered interfaces of the grains. Unfortunately, in snow the phase state is dependent on the stress conditions as well as the temperature. The usual process of bond fracture at the interface occurs. This can be due to a combination of change of properties as well as the stress level. Healing may also happen. Thus, on the interface in the snow material a continuous process of healing and fracture, associated with regelation, must be expected.

The previous discussion suggests that once steady thermal and mechanical conditions exist near the interface, uniform glide speed should be attained. Until that steady state, the glide speed will depend on the presence of water, the organic layer, the snow condition, the slope and ground smoothness. Whether the glide accelerates to the terminal speed, as in the Davos work, or decelerates as in the work shown in figure 1, depends on these local conditions. The final steady speed will also be affected by the local conditions.

A conclusion concerning glide is therefore that a terminal steady speed will be attained once steady mechanical and thermal conditions exist. The prior speed will be closely associated with local characteristics.

A point with regard to the interpretation of field measurements of glide is worth noting. The glide velocity measured at a particular location depends not only on the interface conditions at that point but also on conditions at other locations on the moving interface, on the snow properties, and on the boundary conditions over the whole slope. Thus, meaningful comparison of results obtained at different sites and different geographic locations is particularly difficult.

## Creep

This term is usually applied to deviatoric as well as dilational changes with time. The first is associated with the grains riding over each other, the second with grain readjustment toward a minimum bulk volume. Both processes are aided by the crystals tending to a spherical shape by loss of crystal branches due to vapor diffusion through the air spaces. The grain size decreases and the regime changes allowing local relocation and sliding to proceed rapidly. In spring time the process is aided by the intrusion of melt water from the surface. On a typical slope the creep occurs as motion parallel to and normal to the surface. Shear creep is manifested when the parallel velocity depends on the position of the snow. Dilational creep appears as settlement and uniform bulk motion down the slope. The strain rates in the two modes are of the same order of magnitude on avalanche slopes.

Figure 2 shows results of sawdust column tests in the Cascades. It is noted that the columns remain straight and that the shear creep strain rate is independent of position. This means that the shear strain is constant over the depth whereas the shear stress increases with depth; the constitutive law for the creep process of the snow is not that of a linear, homogeneous, isotropic material.

The creep speeds are of the same order as the measured glide speeds. Both features are therefore equally important in any analytical formulation.
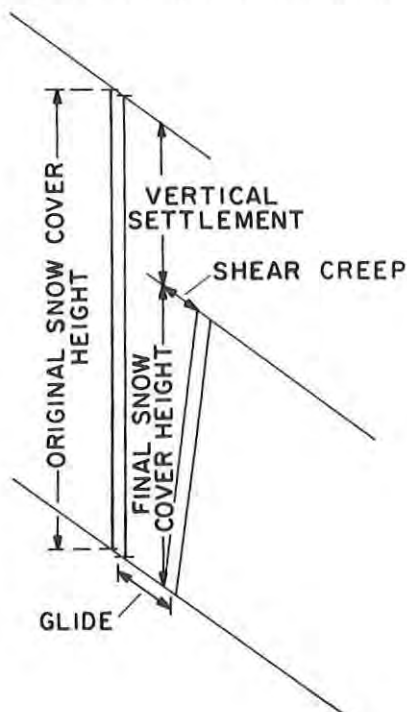


Figure 2. Typical creep experiment, Mt. Baker Site 3.

DURATION OF EXPERIMENT:
  5.087 x $10^6$ sec ($\approx$ 2 MONTHS)
SHEAR CREEP: 1.497 x $10^{-6}$ cm/sec
GLIDE: 0.2295 x $10^{-6}$ cm/sec
ORIGINAL HEIGHT: 422 cm
FINAL HEIGHT: 156 cm
SLOPE ANGLE: 30°, SOUTH-SOUTHWEST FACING SLOPE
TERRAIN: SHORT BRUSH AND GRASS

## Glide Material

The glide boundary condition occurring with thermal and mechanical equilibrium appears to result in a uniform glide speed. At any time the total normal interface area, A, will be comprised of fluid and solid parts ($A_F$ and $A_S$), thus

$$A = A_F + A_S \qquad [4]$$

The area $A_F$ may in fact include separated interface regions discussed by Lang and Brown (1973). The only requirement is that $A_F$ is unable to sustain shear stresses; then the apparent shear stress during glide is

$$\tau = \frac{\tau_s A_S}{A} \qquad [5]$$

where $\tau_s$ is the shear strength of the snow in the region $A_F$. This has a likeness to the Bowden-Tabor (1958) hypotheses. The area $A_S$ will depend not only on the melting and freezing on the interface but also on the snow column weight. This affects the normal contact force on the sintered material considered here. An increase in normal force increases the contact area ($\overline{A}_S$) and results in a higher tangential force to cause incipient rigid body slip. Thus if N is the normal contact force, T the tangential force at the contact between two spheres at incipient slip, then

$$T = \mu_s N \qquad [6]$$

and

$$\tau_s = \frac{T}{\overline{A}_S} \qquad [7]$$

Because the increase in $\overline{A}_S$ with N is nonlinear, it is apparent that the value of $\tau$ in [5] is not only sensitive to the thermal state but also depends on the weight of the snow overburden. The importance of these aspects will vary from site to site.

A steady situation later in the season would be expected to produce an isothermal state, non-modifying snow properties and fixed snow overburden. A possible relationship at this steady state between the interface apparent shear, $\tau$, and the finite glide speed v is

$$\tau = B v^{-1} \qquad [8]$$

where v depends on $A_S/A$ and B on the local conditions providing the values in [6] and [7].

## Creep Model

Consideration of the material suggests that tractable problems in snow mechanics are generally of two types: (1) Those in which disturbances are of short duration and behavior is essentially elastic, and (2) those where loads are sustained for longer times and viscous flow is significant compared to initial elastic motions. For type (1) indications are that at least for low stresses a linear elastic law may be justifiable (Brown et al. 1972). For type (2), consideration of the material as either nonlinear or inhomogeneous lead to motions consistent with those observed. These two forms of steady creep law are investigated here.

For this purpose we will assume uniform slope and snow depth and constant snow density, $\rho$. Coordinates are as shown in figure 3 where an infinite extent in the $x_2$ plane exists. Then the known stress components, $\tau_{ij}$, (Brown et al. 1972) are:

$$\tau_{12} = \rho \sin\Theta \, x_2 \qquad [9a]$$

$$\tau_{22} = -\rho \cos\Theta \, x_2 \qquad [9b]$$
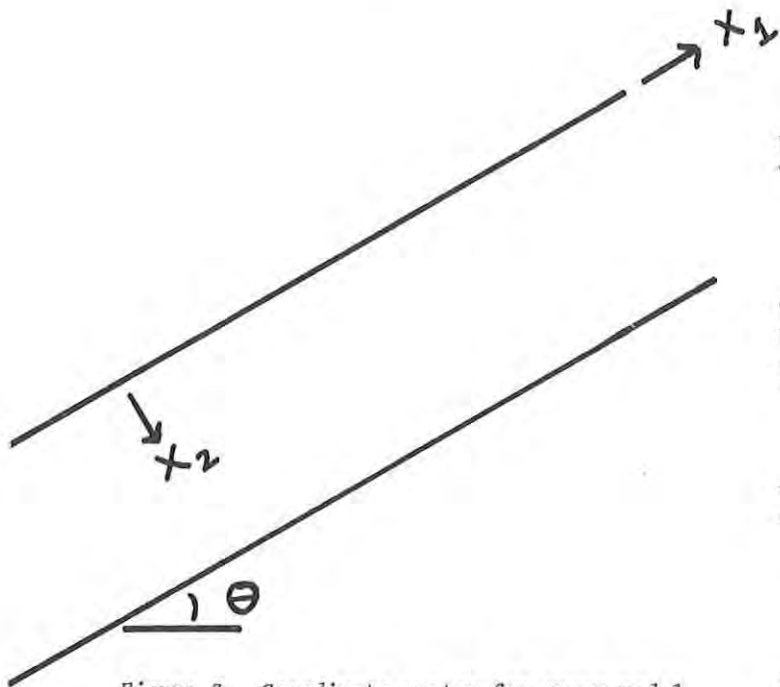
$$\tau_{13} = \tau_{23} = 0 \qquad [9c]$$

Known strain rate components, $d_{ij}$, are:

$$d_{12} = k_1 \qquad [10a]$$

$$d_{11} = d_{33} = d_{13} = d_{23} = 0 \qquad [10b]$$

based on our experimental evidence (figure 2), we assume $k_1$ is constant.

Thus only $d_{22}$, $\tau_{11}$, and $\tau_{22}$ are unknown.

(a) Nonlinear Isotropic Constitutive Law.--
To model an isotropic homogeneous relation between stress and strain rate for steady creep, we assume a constitutive law of the form (Salm 1967):

$$\tau_{ij} = F_1 \delta_{ij} + F_2 d_{ij} \qquad [11]$$

where $F_1$ and $F_2$ are functions of the independent strain rate invariants $I_1$ and $I_2$ and where

$$I_1 = d_{kk} \qquad [12]$$

$$I_2 = \frac{1}{2} d_{ij} d_{ij} \qquad [13]$$

From [10], for the state of deformation under consideration,

$$I_1 = d_{22} \qquad [14]$$

$$I_2 = d_{22}{}^2 + 2k_1{}^2 \qquad [15]$$

From [11]

$$\tau_{12} = F_2 k_1 \qquad [16]$$



*Figure 3. Coordinate system for creep model of long uniform slope.*

which, using [9a] and [10a], give

$$F_2(d_{22}, \ d_{22}{}^2 + 2k_1{}^2) = \frac{\rho \sin\theta}{k_1} \ x_2 \qquad [17]$$

Clearly, the dependence of $\tau_{22}$ on $x_2$ must be known before proceeding further. This information is not presently available, however, and the general relation $\tau_{22}(x_2)$ will be assumed. It may readily be shown that [17] cannot be satisfied if $d_{22}$ is constant. Assuming

$$d_{22} = k_2 \ x_2 \qquad [18]$$

where $k_2$ is constant, then [17] can be most simply satisfied if

$$F_2(I_1, I_2) = \alpha I_1 \qquad [19]$$

where $\alpha$ is a material constant.

Satisfaction of [9b] places restrictions on $F_1$ which using [11] become

$$-\rho \cos\theta \ x_2 = F_1 + \alpha(k_2 \ x_2)^2 \qquad [20]$$

[20] is most simply satisfied if

$$F_1(I_1, I_2) = \beta I_1 - \alpha I_1{}^2 \qquad [21]$$

11

and hence the simplest form of [11] consistent with observations is

$$\tau_{ij} = (\beta d_{kk} - \alpha d_{kk}{}^2)\delta_{ij} + \alpha d_{kk} \, d_{ij} \qquad [22]$$

and in terms of $\alpha$ and $\beta$ the parameters of deformation, $k_1$ and $k_2$, are given by

$$k_1 = -\frac{\beta}{\alpha} \tan\theta \qquad [23]$$

$$k_2 = -\frac{\rho \sin\theta}{\beta} \qquad [24]$$

(b) Inhomogeneous Constitutive Law.--The observed shearing deformation will occur if

$$\tau_{12} = 2\Sigma d_{12} \qquad [25]$$

where $\Sigma$ is a material constant which increases linearly with depth and is zero on the upper surface. Such inhomogeneity would result if the flow law for steady creep were of the form

$$\tau_{ij} = \lambda\delta_{ij} + 2\Sigma \, d_{ij} \qquad [26]$$

where $\lambda$ and $\Sigma$ are material constants which depend on invariants of the state of strain, that is, stress-induced inhomogeneity. The observed deformation would occur if $\Sigma$ were linear in volumetric strain (or hydrostatic stress).

Argument (b) for inhomogeneous material provides a simpler explanation of the observed shearing motion than argument (a). For this reason, it must be preferred at this stage. Provided that $\lambda$ and $\Sigma$ are strain or stress dependent, then [26] still describes nonlinear material behavior. For some stages of creep, however, where the state of stress remains constant, the behavior will be equivalent to that of an inhomogeneous isotropic material. Clearly, further experimental evidence is required before more definite conclusions may be drawn with regard to the constitutive law for steady creep.

## Avalanche Prediction

From the hypothesis of Perla and LaChapelle (1970) and the subsequent full analyses (Gand and Zupancic 1965), it is clear that one avenue of avalanche prediction concerns the attenuation of basal shear capacity. The regelation model suggested here for the steady speed case would mean that over a substantial total interface area A some part will be melted and some frozen. Additionally, separation may occur. Under these circumstances [4] and [5] apply. Over $A_F$ the shear capacity will be zero whereas over $A_S$ the capacity at incipient sliding will be $\tau_s$. This will reduce when a value $\mu_k \leq \mu \leq \mu_s$ is introduced into [6]. Averaged over the interface, this will account for some regions slipping locally and some just on the point of slipping. A consequence is that as $A_S/A$ decreases, the apparent shear capacity, $\tau$, decreases and the glide speed increases. Thus the physical model for glide resistance of Gand and Zupancic (1965) and by figure 1 can be explained by the hypothesis of [5]. In this case, our interest is in the sense of the ration $A_S/A$ and hence in the sign of the rate of change of glide speed with respect to time. When the glide speed is increasing, with no additional evidence of increase in snow strength, then conditions of incipient avalanching should be expected. With decreasing glide speed, the slope should be stabilizing. In figure 1 the increasing glide speeds in February forecast periods of danger because of the increase in $A_F$; the decreasing speeds in February and March give rise to confidence in the snow slope stability.

## Acknowledgments

12

## Literature Cited

Bowden, F. P., and D. Tabor.
  1958. The friction and lubrication of solids. Oxford Univ. Press.
Brown, C. B., R. J. Evans, and E. R. LaChapelle.
  1972. Slab avalanching and the state of stress in fallen snow. J. Geophys. Res. 77:4570-4580.
Gand, H. R., and M. Zupancic.
  1965. Snow gliding and avalanches. Int. Symp. Sci. Aspects of Snow and Ice Avalanches, Davos, Switz., p. 230-242.
Lang, T. E., and R. L. Brown.
  1973. On the mechanics of the hard slab avalanche. p. 24-28. In Advances in North American avalanche technology: 1972 symposium. USDA For. Serv. Gen. Tech. Rep. RM-3, 54 p. Rocky Mt. For. and Range Exp. Stn., Fort Collins, Colo.
Perla, R. I., and E. R. LaChapelle.
  1970. A theory of snow slab failure. J. Geophys. Res. 75:7619-7627.
Rowe, G. W.
  1964. Friction and metal-transfer of heavily-deformed sliders; mechanisms of solid friction. Elsevier Pub. Co., N.Y., p. 204-216.
Salm, B.
  1967. An attempt to clarify triaxial creep mechanics of snow. In Physics of snow and ice, Hirobumi Ôura, Ed. Int. Conf. Low Temp. Sci. [Sappora, Japan, Aug. 1966] Proc., Vol. I, Part 2, p. 857-874. Inst. Low Temp. Sci., Hokkaido Univ.